International Journal of Control, Energy and Electrical Engineering (CEEE)
Vol.8 pp.7-11

Linear-Quadratic-Gaussian control design for an Hydraulic system

Mounir BEKAIK#1, Abdelghani REDJATI*2

#1,2Annaba University, Laboratory of LASA
Algeria

[1]mounir.bekaik@gmail.com
[2]redjati@yahoo.fr

***Abstract*— This work is devoted to the stabilization of a family of multi-model systems. We have chosen to focus on the application of Gaussian linear quadratic control on a three tank hydraulic system. This controller is based on the LQ quadratic linear control as well as the famous Kalman observer filter. On the other hand, the application of this control is a complicated task because of the nonlinearities and the presence of defects. In order to apply our control strategy, it consists to represente the non-linear system as a set of several linear submodels through a new technique based on Fuzzy C-Means and N4Sid identification algorithm.**

Thanks to the Takagi Sugeno representation, the control of the system is obtained by combining the stabilizing control laws of linear subsystems.

***Keywords*— LQG, Multi-model, Takagi-Suegno, Fuzzy.**
.

I. Introduction

. Optimal control problems [2] are encountered in everyday life: how to get to a destination as quickly as possible, how to minimize consumption. For a given dynamic system whose equations are known, the optimal control problem [7] is then to find the control that minimizes a given criterion. The LQG control [7] has the advantage of applying to systems whose condition is not fully measured. Developed at the beginning of the second half of the 20th century, it emerged as the first general method for the control of multivariable systems [1], [3]. Since the 20th century, many publications have been published testifying to the success of the LQG order.

The main contribution of our work is to propose a Gaussian quadratic linear control for an hydraulic system which is a highly nonlinear system. The development of these new results is based on the Multi-model approach [4].

This controller is developed from a multi-model representation of the Takagi-Sugeno [9] form of the system. For each linear model on the system reference path, an optimal multivariable LQG control law [11], [8] is established minimizing a quadratic criterion depending on the different control objectives such as uncertain additive noise. The control applied to the system is then obtained by interpolating the control laws of the different linear subsystems.

Takagi-Sugeno approximation [10] that relies on a bank of piecewise linear models to capture the possible input-output response behavior has been developed. Using a divide and conquer strategy, local linear models set is described and the global output is obtained by the integration of locale ones. Our approach is based on the fuzzy C-Means (FCM) [5], [6] . This algorithm is used to find operators regimes which are associated to dynamical linear local models. The clusters are formed according to the distance between data points and the cluster centers are formed for each cluster through N4sid identification approach. The N4SID algorithm [12] allows modeling a system from the measured input and output data. it leads to determining the order of the system by applying the dominant singular value technique.

One of the main advantages of the N4SID is that it is non-iterative and does not require the involvement of non-linear optimization methods. This allows it to overcome the problems exposed when applying iterative techniques that suffer from the absence of a guarantee of convergence and minimization of the criteria mentioned and from the sensitivity to the estimation of the initial state.

These features make the implementation of such method provide a system state representation that facilitates the implementation of the LQG control.

This paper is organized as follows. Section 2 is devoted to the Fuzzy approach and Takagi-sugeno representation. In Section 3 we present our main result in Section 4 which introduces the TS fuzzy system subject to define multi-model LQG controller. An application of the results is made on a hydraulic system Section 5. In the end, a conclusion will be quoted in Section 6.

II. FUZZY IDENTIFICATION APPROACH

A. Fuzzy C-Means algorithm:

The first step consists to dividing data elements into classes or clusters using FCM algorithm. This algorithm is used for analysis based on distance between various input data points. The clusters are formed according to the distance between data points and the cluster centers are formed for each cluster. In fact, FCM is a data clustering technique in which a data set is grouped into n clusters with every data point in the dataset related to every cluster and it will have a high degree of belonging (connection) to that cluster and another data point that lies far away from the center of a cluster which will have allow degree of belonging to that

ISSN 2356-5608

cluster.

Algorithmic steps for Fuzzy C-Means.

We are to fixe *c* where *c* is ($2 \le c \le n$) and then select a value for parameter "m" and there after initialize the partition

1) We calculate the center for each cluster

$$v_{ij} = \frac{\sum_{K=1}^{K} (u_{ik})^m x_{kj}}{\sum_{n=1}^{N} (u_{ij})^m} \quad (1)$$

2) The distance matrix $D_{[c,n]}$ is given by:

$$D_{ij} = \left(\sum_{j=1}^{m} (x_{kj} - v_{ij})^2\right)^{1/2} \quad (2)$$

3) Update the partition matrix for the r^{th} step $u_{ij}^{r-1} = \frac{1}{\sum_{j=1}^{c} \left[d_{ik}^{r} / d_{ik}^{r} \right]^{\frac{2}{m-1}}}$ (3)

If

$$\left\| U^{it+1} - U^{it} \right\| < \zeta , \quad (4)$$

then we are to stop otherwise. We have to return to step 2 by updating the cluster centers iteratively and also the membership grades for the data point.

The second step consists to define for each cluster a local model using Subspace identification approach N4SID .This algorithm is based on the estimation of the state sequence matrix to estimate the matrices of the system and Kalman gain. One of the major advantages of N4SID is that it is non-iterative and does not involve non-linear optimization methods. This allows it to overcome the problems presented by the application of iterative techniques that suffer from the unsecured convergence and the minimization of the target criterion as well as the sensitivity to the estimation of the initial state . These features make the implementation of such a method attractive and provide a system state representation facilitating the implementation of the LQG command.

B. Fuzzy Takagi-Sugeno presentation:

A nonlinear dynamic system can be described in a simple way by a Takagi-Sugeno fuzzy model, which uses series of locally linearized models from the nonlinear system. So any TS fuzzy model of a nonlinear system is structured as an interpolation of linear systems. The i-th rule is expressed as

If Z_1 is F_i^1 and $z_p(t)$ is F_i^p Then:

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i u(t) \\ y(t) &= C_i x(t) \end{aligned} \quad (5)$$

F_i^j are fuzzy sets. $x(t) \in R^n$ is the reference state, $u(t) \in R^m$ is the control, $y(t) \in R^q$ is the reference output, $A_i \in R^{n*n}$ $B_i \in R^{n*m}$, $C_i \in R^{q*n}$ and $z_1(t)...z_p(t)$ is a known vector of premise variables. Given a pair of (*y*(*t*), *u*(*t*)) the fuzzy system inference is given by:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(z(t))(A_i x(t) + B_i u(t))}{\sum_{i=1}^{r} w_i(z(t))} \quad (6)$$

$$y(t) = \frac{\sum_{i=1}^{r} w_i(z(t)) C_i x(t)}{\sum_{i=1}^{r} w_i(z(t))} \quad (7)$$

With: $z(t) = \begin{bmatrix} z_1(t) & z_2(t) & \cdots & z_p(t) \end{bmatrix}$, (8)

$w_1(z(t)) = F_i^j(z_j(t)) \quad i = 1,2,...,r$

where $w_i(z(t))$ are normalized rule firing strengths, and $\forall t \ge 0$ we have:

$$\begin{aligned} &\sum_{i=1}^{r} w_i(z(t)) \succ 0 \\ &w_i(z(t)) \ge 0 \end{aligned} \quad (9)$$

Consider :

$$h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))} \quad (10)$$

T-S fuzzy model can be inferred as:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)) \\ y(t) &= \sum_{i=1}^{r} h_i(z(t)) C_i x(t) \end{aligned} \quad (11)$$

The rule firing strengths $\sum_{i=1}^{r} h_i(z(t)) \ge 0$, verify :

$$\sum_{i=1}^{r} h_i(z(t)) = 1 \quad (12)$$

An approach to obtain T-S fuzzy model that has been used in this work is local approximation in fuzzy partition spaces. In

ISSN 2356-5608

fact, in this method, nonlinear terms have been approximated by chosen linear terms. This procedure leads to reduction of the number of model rules. The number of model rules is directly related to complexity of analysis and design LMI conditions or Riccati equation solution. This is because the number of rules for the overall control system is basically the combination of the model rules and control rules

III. MULTIMODEL LQG CONTROL DESIGN

Consider the system (13) such that the output and the state are disturbed by noise

$$\dot{x}(t) = A\,x(t) + B\,u(t) + w_1(t)$$
$$z(t) = C\,x(t) + w_2(t) \qquad (13)$$

with: *dim x(t) = n x 1, dim u(t): l x 1,* where *l* is the number of actuators, *dim z(t) = m x 1*

$w_1(t)$ and $w_2(t)$ are two vectors of Gaussian white noise where *dim* $w_1(t)$ *:n x l* and *dim* $w_2(t)$ *:m x l.*

with $w(t) = \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$, where the noises $w_1(t)$ and $w_2(t)$ are independent and stationary, we obtain:

$$E\{w(t)w^T(\tau)\} = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} \delta(t-\tau) \qquad (14)$$

The matrix v= $\begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}$ is a constant variance covariance matrix, where the matrices v_1 and v_2 are positive definite symmetric.

The LQG regulator consists of two parts: an optimal state-feedback gain and a Kalman state estimator.

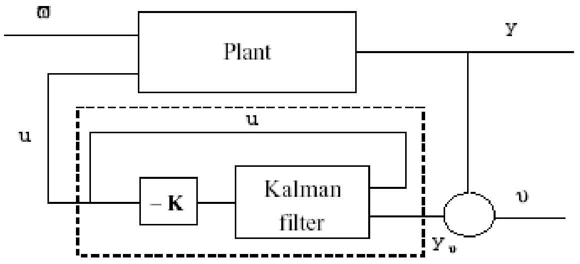

Fig. 1 LQG controller

The feedback gain matrix is sought to minimize a quadratic performance criterion *J* expressed as:

$$J_{LQG} = E\{\int_0^\infty [x^T(t)Q\,x(t) + u^T(t)R\,u(t)]dt\} \qquad (15)$$

The weighting matrices Q and R are positive definite matrices.

The gain of the feedback $u(t) = -Kx(t)$ that minimizes the cost function J is:

$$K = R^{-1}B^T P \qquad (16)$$

is usually called the LQ-optimal gain, where the matrix *P* is obtained by solving an algebraic Riccati equation:

$$P\,A + A^T P + Q - P\,B\,R^{-1}B^T P = 0 \qquad (17)$$

The next step is to derive a state estimator $x(t)$ generated by Kalman filter. The gain L of the observer is given by

$$L = \tilde{Q}C^T V_2^{-1} \qquad (18)$$

Where Q is the solution of the following Riccati equation

$$A\,\tilde{Q} + \tilde{Q}A^T + V1 - \tilde{Q}C^T V_2^{-1}\,C\,\tilde{Q} = 0. \qquad (19)$$

Through TS representation, the fuzzy LQG controller is given by:

$$u(t) = \sum_{i=1}^{r} h_i(z(t))u_i(t) \qquad (20)$$

Where $u_i(t)$ is the LQG controller for each subsystem.

IV. APPLICATION AND SIMULATION

The nonlinear controlled system consists of three cylinders T1, T2 and T3 with the cross-sectional area A which are interconnected in series by two connecting pipes. The liquid (distilled water) leaving T2 is collected in a reservoir from which pumps 1 and 2 supplies the tanks T1 and T2. All three tanks are equipped with piezo-resistive pressure transducer for measuring the level of the liquid. A digital controller controls the flow rate Q1 and Q2 such that the levels in the tanks T1 and T2 can be preassigned independently. The level in tank T3 is always a response which is uncontrollable. The connecting pipes and the tanks are additionally equipped with manually adjustable valves and outlets for the purpose of simulating clogs as well as leaks.

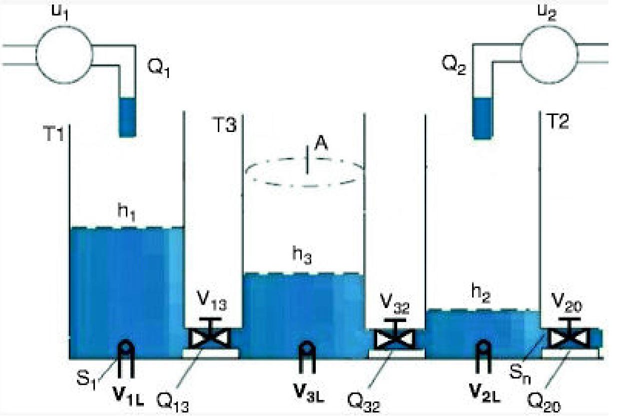

Fig. 2 Three thank hydraulic system

ISSN 2356-5608

The flow rates q_1 and q_2 are defined through the rotational flow. The interconnection rates Q_{13} et Q_{32} depend of levels on the tanks. The flow rate Q_{20} is the output of the system. The input vector is given by:

$$U = [Q_1 \quad Q_2]^T \tag{21}$$

Then, the output vector is given by:

$$Y = [h_1 \quad h_2 \quad h_3]^T \tag{22}$$

The selected nonlinear system composed of three tank system, have a measurement data of inputs and outputs. Due to its high nonlinearity, and inaccessibility of some its outputs and states for measurements, the system is often perceived as a challenging engineering problem. In order to control the levels in the tanks, the following steps are also made: The first step consists to find the local linear models for TS fuzzy model through Fuzzy C-means and N4sid approach.

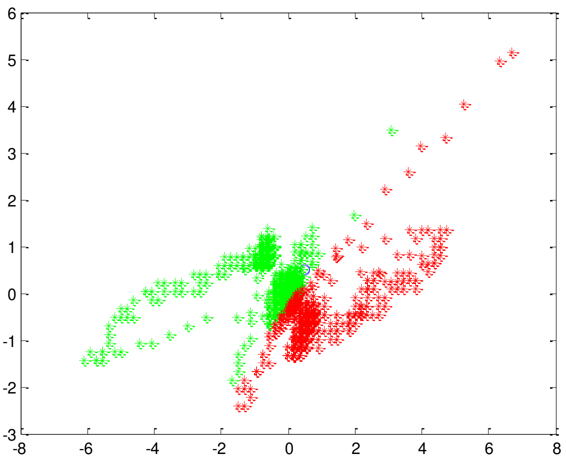

Fig. 3 fuzzy C-means decomposition.

According to Fig. 3, Fuzzy C-means is a suitable method for approximation of this system, with this approximation the nonlinear and linear parts of the system. Due to the decomposition to two regions, we can utilize a LQG controller for each local linear model.

Then, the second step consists in the design of a LQG controller, which stabilizes the nonlinear fuzzy system.

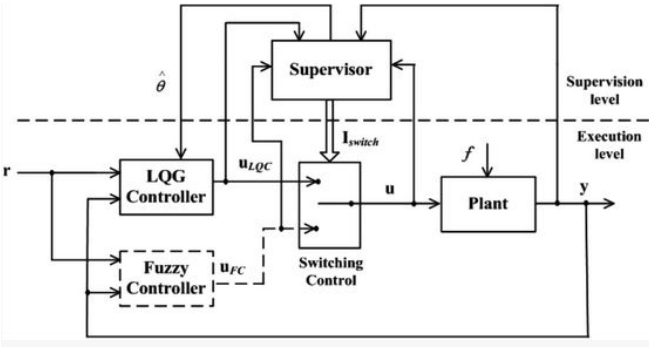

Fig. 4 Fuzzy LQG controller

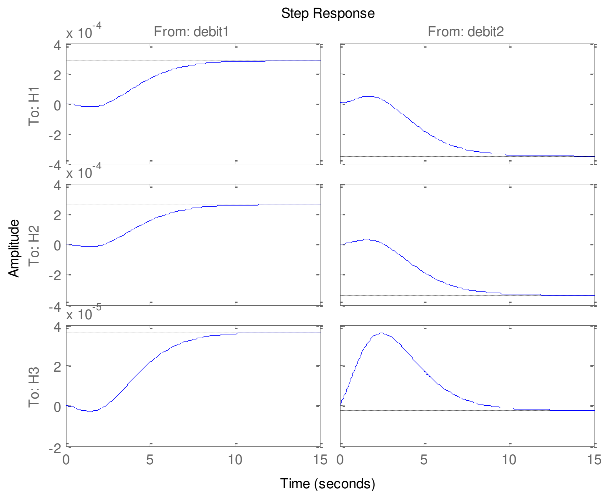

Fig. 5 Time response of the system with the fuzzy LQG Controller

According to Fig 5 the LQG control provides very good performance in terms of good reference tracking, estimated response time less than 10 s and there is also no static error. LQG controller keeps this stability with different variance of the flow rates.

V. CONCLUSIONS

In this work, we proposed Gaussian linear quadratic control for a three tank hydraulic system. To handle the problem caused by nonlinear data, we choose Fuzzy c-means algorithm and N4SID identification approach to define local linear models. The Fuzzy LQG control is based on TS presentation of the system. The simulations show the effectiveness of our approach, the fuzzy LQG control ensures the stabilization of the system. One topic for future research may be in the use of the subspace identification approach to define local linear models. We can use an extended Kalman filter to define LQG control and compare it with a multi-model adaptive control.

REFECRENCES

[1] A. Boulkroune, M. Saad and M. Farza, « *Adaptive fuzzy controller for* Multivariable nonlinear state time-varying delay systems subject to input nonlinearities ». Fuzzy Sets and Systems, In Press 2010.

[2] D. Subbaram Naidu, « *Optimal Control Systems* ». CRC Press, 2003

ISSN 2356-5608

[3] E. Ostertag, « *Commande et Estimation Multivariables* », TechnoSup, Ellipses, Paris, 2006.

[4] G. Li and F. Zhao. « Flexibility control and simulation with multi-model and LQG/LTR design for PWR core load following operation». Annals of Nuclear Energy, Volume 56, June 2013.

[5] J. Abonyi , R. Babuska and F. Szeifert, « *Modified Gath-Geva fuzzy clustering for* identification of Takagi-Sugeno fuzzy models ». IEEE Transactions on Systems, Man and Cybernetics, Part B: Cybernetics, 32(5):612–621.2002.

[6] V. S. Rao and Dr. S. Vidyavathi, «Comparative Investigations and Performance Analysis of FCM and MFPCM Algorithms on Iris data », Indian Journal of Computer Science nd Engineering.

[7] B. Cardenas and M. Molinas, « Optimal LQG Controller for Variable Speed Wind Turbine based on Genetic Algorithms », Energy Procedia, vol. 20, pp. 207–216, 2012.

[8] S. Nourdin, H. Camblong, I. Vechiu and G.Tapia, « Comparison of wind turbine LQG controllers using individual pitch control to alleviate fatigue loads », IEEE conference on control & automation, pp. 1591-1596, 2010.

[9] T. Johansen, Shorten and R Murray-Smith, « On *the interpretation and* Identification of dynamic Takagi-Sugeno fuzzy models ». IEEE Transactions on Fuzzy Systems, 8(3):297–313.2000.

[10] T. Takagi and M. Sugeno, « Fuzzy identification of systems and its application to *modeling and control* ». IEEE Transactions on Systems, Man and Cybernetics, 15:166–172.1985.

[11] X. Yang and O. Marjanovic, « LQG Control with Extended Kalman Filter for Power Systems with Unknown Time-Delays ». preprints of the 18th ifac world congress, Milano (Italy) August 28 - September 2, 2011.

[12] P. Van Overschee, and B. De Moor, N4SID: « Subspace algorithms for identification of combined Deterministicstochastic Systems », Automatica, , Vol. 30, pp. 75-93,1994.

ISSN 2356-5608